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OPTIMIZATION OF PLANNING IN RAILWAYS USING PROPOSED MATHEMATICAL MODEL

ALIASGHAR ARAB DERASELEH* AND MAHMOUD R. KEYMANESH

Payam Noor University

***Corresponding Author: E Mail: Arabmeysam9@gmail.com**

ABSTRACT

Planning for passenger lines is a strategic and long-term decision in field of passenger planning in railways. Various mathematical models have been proposed and applied for planning to date. At the present study, a solution has been presented on basis of column generation algorithm, in which subject of passenger line planning has been defined as a main problem and two sub-problems. As obtained result from column generation method is other than integers, an innovative algorithm has been proposed for purpose of changing obtained results into integers. Target function of passenger line planning in this paper is maximizing number of direct passengers. Validation of proposed solutions has been conducted using 12 sample problems and comparing obtained results from the proposed method through solution based on branch and limit.

Keywords: Railway Transportation Planning, Passenger Line Planning, Optimization, Mathematical Model

INTRODUCTION

Passenger line planning is a strategic and long-term decision in filed of passenger planning in railways. Various mathematical models have been proposed and applied for planning so far. Movement of different trains in a single line has certain destinations and different times of dispatch and arrival. The aim of the desired problem is selecting optimized lines from proposed lines, which provide maximum services for passengers and can also minimize operative costs for the lines. Supplied services for passengers can be described using several indices. One of the main indices is direct passenger. Usually, passengers tend to go through their rout directly from source to destination. Direct passengers in line planning are considered as passengers, who have no need for switching their line in their trip from source to destination.

Operative costs are usually adjusted with length and number of lines. Required inputs for line planning problem include existing infrastructures and trip demand between every source-destination couple. Therefore, demand would be defined as a source-destination matrix. If rail network is presented as G (V, E), V can present stations and E is a sign for routs between two assumed frequent stations or same network arcs. In regard with line planning problem, only those stations are considered that have minimum number of passengers. Hence, stations that have porterage use or those stations that have minimum passengers are not considered. Set V is formed of n stations or nodes. Matrix of input demand is n×n matrix. It should be noted that in this model, demand in the trip and demand in return trip are considered similar and they would be assumed equal to maximum demand for going or returning. The reason for this problem is circular use of passenger cars (wagons). It means that the train returns in reverse rout of its trip. Hence, number of iteration in trip rout should be similar to number of iteration in return rout. Hence, maximum demand for round trip can be considered as coupled demand of the station. Passengers in source may have different routs for achieving destination; although it is assumed in all relevant models that passengers choose the shortest rout to arrive their destination. In addition, they can choose passenger lines in a manner that they can have stop in all stations or have in specified stations for same line due to applied type of train **[1]**.

Results of scheduling moving times of trains are dispatch and arrival times of each train in every station. Hence, such planning is so important in railway transportation planning. Results of scheduling for trains movement can be an input for scheduling vehicles. In this step of planning, due to arrival and dispatch times of trains, allocation of navigation to each dispatch would be conducted. The last step is servers planning. In this step, work program of servers of trains would be determined in a manner that the minimum costs would be allocated through observance of work regulations **[2, 3]**.

LITERATURE REVIEW

Demand prediction is the first step toward railway planning in scope of transporting passenger, which can be considered as one of the main prerequisites of line planning or determining passenger routs. In this step, demands between every couple of stations would be met. Exact prediction of passenger demand can be a basis for other plans in passenger transportation. Numerous studies have considered passenger demand forecasting, among which one can refer to Tsai *et al* (2009) **[4]** and Suryani *et al* (2010) **[5]**. The ext step is passenger line planning, in which optimized lines with certain iterations would be obtained for each line. One of the most important relevant works is study of Bussieck *et al* (1996) on maximizing number of direct passengers **[6]**. The mentioned study has assumed capacity of all passenger trains fixed and same and has also presented different methods for purpose of solving its model.

Claessens *et al* (1998) have presented an integer non-linear model for purpose of finding optimized lines in order to minimize operative costs of the lines **[7]**. In addition, in order to solve the model, it has been changed into an integer linear model using optimization software. Goossens *et al* (2004) has also presented a model for minimization of line costs and have solved the model using a branch and cut algorithm **[8]**. Lindner (2000) has also studied minimization of railway operative costs **[9]**. The author has presented a branch and bound method for purpose of solving the model and has also integrated it with train schedule model.

Scoll (2005) has investigated minimization of number passengers, who need to change their line to arrive their destination **[10]**. The study indicates that minimization of number of the passengers is more complicated that maximization of direct passengers and that Lagrange release should be applied for purpose of finding a low level.

Several studies have been also presented recently with topic of passenger line planning in railway, among which several studies are as follows. Shafia *et al* (2010) have presented a mixed integer model through considering uncertainty in input information of problem with passenger line planning and used a heuristic algorithm to solve it **[11]**. Yaghini *et al* (2012) have used goal planning technique in order to present a multipurpose to a one-purpose model in order to plan for passenger transport **[12]**. In addition, Yaghini et al (2012) have also presented a hybrid method for solving passenger railway problem using analysis algorithm and presented than a heuristic method **[13]**. Results of passenger railroad scheduling are input for train scheduling.

Proposed mathematical model

Mathematical model of passenger line scheduling used in this study can maximize number of passengers among all sources and destinations. Limitations with passenger line scheduling include limitation of demand, limitation of provided capacity and limitation of total number of train dispatch on every arch of the network.

Counters

e: counter of network arcs $e \in E$

r: counter of lines $r \in R$

a, b: counter of source and destination station $a, b \in V$

Set

V: set of available stations

E: set of network arcs

R: set of offered lines

Parameters

$d_{a,b}$: Demand for trip from a to b

$u(e)$: High limit of iteration through dispatching train on the arc e

c: Capacity of navigation

Decision variables

$y_{r,a,b}$: decision variable in kind of integer; presenting number of direct passengers from source "a" to destination "b" by the line "r"

x_r: Decision variable for the problem in kind of integer; presenting number of iteration of line "r"

$$Max \sum_{a,b \in V} \sum_{e \in r \; r \in R_{a,b}} y_{r,a,b} \qquad (1)$$

$$\sum_{e \in r \; r \in R_{a,b}} y_{r,a,b} \leq d_{a,b} \qquad \forall a,b \in V \qquad (2)$$

$$\sum_{e \in r \; r \in R_{a,b}} y_{r,a,b} \leq c\, x_r \qquad \forall r \in R, e \in r \qquad (3)$$

$$\sum_{e \in r \; r \in R_{a,b}} x_r \leq u(e) \qquad \forall e \in E \qquad (4)$$

$$x_r \geq 0 \;\&\; Integer, \;\; y_{r,a,b} \geq 0 \;\&\; Integer. \qquad \forall r \in R, \forall a,b \in V \qquad (5)$$

Target function (1) maximizes total number of direct passengers, who arrive in their destination through all lines. Limitation (2) indicates that total number of passengers, who travel directly between each source and destination, should be less than demand for the trip. Limitation (3) states that in each line, total number of direct passengers in a specific arc should be less than capacity created by dispatching the train. Limitation (4) indicates also that total dispatches of lines on each arc should be less than high limit of dispatch on same arc. The high limit can be obtained due to some factors such as system of signs and communications and number of encountered stations in the arc, which can be considered in this problem as one input parameter. Limitation (5) defines decision variables for the problem, which both types of decision variables are in kind of integers **[14]**.

Proposed solution

Proposed solution in this study is based on column generation algorithm. The algorithm is a careful solution to achieve optimized result for linear planning problem, which is also known as analysis algorithm. Using this method is for solving big problems, which can't be solved by optimization software in desirable time.

In column generation algorithm, the primary problem would be divided to a main problem and at least one sub-problem. In each step of implementing column generation algorithm, dual values, output of the main problem and new values of decision variables can be output for sub-problems. Hence, space of problem solving would be divided at least to two parts. As mentioned, the proposed method is suitable for solving problems with linear planning. Thus, there is no guarantee for achieving integer results in problems that their decision problems are in this group. At the present study, line planning problem is divided to one main problem and two sub-problems. In order to change obtained results to integers, a heuristic algorithm has been presented. In each step of algorithm, dual values would be putted into sub-problem from the main problem and in sub-problem, new values would be put into the main problem. The condition for ending the algorithm would be investigated by target function of the sub-problem. Target function of the primary problem is same target function of the main problem. As mentioned before, the method can change the problem into a main problem and at least one sub-problem.

In order to solve passenger line planning problem by column generation method, limitation (3) that includes both variables of number of direct passengers and number of line dispatches is put into the main problem and two sub-problems are defined for purpose of realizing other limitations. The first sub-problem can establish limitation (2) and second sub-problem can provide limitation (4). Hence, in regard with big problems and due to one main problem and two sub-problems, space of problem solving would be divided to three parts and speed of problem solving would be increased. Main problem would be obtained as follows:

$$Min\ \sum_j (-\sum_{a,b\in V}\sum_{e\in r\ r\in R_{a,b}} y_{r,a,b})\lambda_j \qquad (6)$$

$$\sum_j ((\sum_{e\in r\ r\in R_{a,b}} y_{r,a,b})\lambda_j + (-c\, x_r)\gamma_j) + s = 0 \quad \forall r \in R, e \in r \qquad (7)$$

$$\sum_j \lambda_j = 1.0 \qquad (8)$$

$$\sum_j \lambda_j = 1.0 \qquad (9)$$

$$\lambda_j \geq 0,\ \ \lambda_j \geq 0\,, s \geq 0. \qquad (10)$$

In model 6-10, target function (6) that is same target function of the primary problem, which has been changed into committing form n order to adjust it with column generation method. Decision variable of the main problem is λ and γ. Limitation (7) is same limitation for provided capacity by dispatches on each arc and vector of variable of shortage (s) changes the limitation into equal form [ibid].

Limitation (8) is limitation of convexity on obtained results from the first sub-problem. Limitation (9) is limitation of convexity on obtained results from second sub-problem. Limitation (10) can define decision variables of the problem in positive form. In 6-10 model, x and y vectors are inputs of the problem from sub-problems and also are considered as model parameters. Vector of dual values would be illustrated for limitation (7) as w; for (8) as α and for (9) it would be depicted as β.

The first sub-problem that establishes limitation (2) of the problem and specifies values of decision variables of y in each stage has been defined as 11-13 models.

$$Max\ ((w \times A) + 1)y + \alpha \qquad (11)$$

$$\sum_{e\in r\ r\in R_{a,b}} y_{r,a,b} \leq d_{a,b} \quad \forall a,b \in V \qquad (12)$$

$$y \geq 0. \qquad (13)$$

Target function (11) determines decision variables of type y and their values for the main problem. As it was mentioned, w is vector of dual values for limitation (7) and A is coefficient for y decision variables in limitation (7). Limitation (12) is same demand limitation in the primary problem. Limitation (13) defines decision variable of the first sub-problem. In regard with passenger line planning, decision variables are in type of integers. However, as in the column generation method space of solving sub-problems should be convex, one can't define decision variables in type of integers. As the variables indicate number of direct passengers and number of them is usually a big number, considering this variable as integer can't change the results. One can simply obtain an integer value through rounding the value [ibid].

Second sub-problem that is responsible for selecting decision variables type x, can be defined as 14-16 models.

$$Max\ (W \times A)x + \beta \qquad (14)$$

$$\sum_{e \in r\, r \in R_{a,b}} x_r \leq u(e) \qquad \forall e \in E \qquad (15)$$

$$x \geq 0. \qquad (16)$$

Second sub-problem can also encompass limitation that includes only x decision variable. Target function (14) selects x decision variables for entering to the base and allocates also adjusted values to these variables. Limitation (15) is same high level limitation of dispatch on each arc and function (16) can also define decision variable positive type. Obtained result from the decision variable by column generation algorithm is non-integer. As the variable presents number of dispatch of each line, it should be in form of integer, which would be changed into integers using heuristic algorithm. The requisite for ending column generation algorithm is as follows:

$$Max\ (W \times A)x + \beta \leq 0 \qquad \&$$

$$Max\ ((w \times A) + 1)y + \alpha \leq 0. \qquad (17)$$

According to (17), column generation algorithm would be stopped, when target functions for both problems are equal or below 0 simultaneously. After ending column generation algorithm, values of dispatch numbers in each line would be put into heuristic algorithm to change it into integer values.

As it was mentioned, obtained results from the column generation method are not surely integers and required results for railway planning problem should be in form of integer. The main output of the problem is number of train dispatch in each line or passenger rout. Hence, non-integer value for this variable is insignificant. In order to change obtained results into integers, a heuristic algorithm has been proposed. Steps for the proposed heuristic algorithm are as follows:

Step 1: total dispatches on each arc would be calculated.

Step 2: total obtained dispatches n the step 1 would be rounded to the nearest integer.

Step 3: amount of dispatches on each arc would be rounded separately and then they would be calculated totally.

Step 4: obtained difference from rounded set of dispatches and rounded number of total dispatches would be calculated.

Step 5: according to obtained difference from rounded set of dispatches and rounded value of total dispatches, lines with longest possible route would be generated on the network.

Evaluation of proposed method

To ensure about obtained results from the proposed method, 30 sample problems with different sizes have been designed according to **Table 1**. In solved problems, number of stations has been considered to 10, 20 and 30. Proposed lines have been also defined in two modes. In the first mode, a proposed line has been defined among all stations, which here is depicted by (F). In second mode, 50% of all proposed lines in the first mode have been defined as potential lines [ibid]. The lines have been selected in a manner that it is possible to displace passengers directly and hence, long lines have been mostly considered. Proposed lines in this mode have been illustrated as (H). In regard with capacity of dispatches on each $u(e)$ arc, two modes have been also considered. In the first mode (T), capacity of dispatch on each arc has been considered in low level and in the second mode (L), high value has been considered for it.

CONCLUSION

At the present study, a base method of column generation algorithm has been proposed for purpose of solving railway planning problem. Passenger line planning model applied in this study includes target function of maximizing number of passengers using limitations of demand, provided capacity by lines and high level of dispatch on each arc. In order to solve the mentioned model using proposed method, limitation of capacity provided by lines is considered as the limitation for the main problem and two sub-problems have been defined for each group of limitations of demand and high level of train dispatch on each arc. To ensure about obtained results from the proposed model, 12 sample problems with different sizes have been designed.

Solving railway planning problem with less than 20 stations is possible using optimization software. However, it could be noted that the time for solving in this software in regard with medium problems is so long using proposed method based on column generation algorithm.

Hence, using optimized software for solving railway planning problems would be proposed only for small size problems. Solution based on column generation algorithm has ability to achieve optimized results for medium and even relatively big problems in reasonable time. Therefore, using the proposed method in such problems can be efficient.

REFERENCES

[1] M. G. P. F. Yaghini, "presentation an innovative model for planning servers and railway personel tasks," Journal of Transportation, vol. 6, no. 4, pp. 381-396, 2009.

[2] C. L. G. Barnhart, "handbook in OR and MS," Elsevier B.V, vol. 14, 2007.

[3] J. W. H. S. V. a. K. L. Goosens, "on solving multi-type railway line planning problem," European Journal of Operational Research, vol. 168, no. 2, pp. 403-424, 2006.

[4] T. H. Tsai and W. C. Lee C.K, "neural network based temporal feature models for short-term railway passenger demand forecasting," Expert Systems with Applications, vol. 36, no. 2, pp. 3728-3736, 2009.

[5] E. Suryani, C. S and C. C.H, "air passenger demand forecasting and passenger terminal capacity expansion: a system dynamics framework," Expert Systems with Applications, vol. 37, no. 3, pp. 2324-2329, 2010.

[6] M. R. Bussieck, K. P and U. Zimmermann, "optimal lines for railway systems," European Journal of Operational Research, vol. 96, pp. 54-63, 1996.

[7] M. Claessens, M. Van Djik N and Z. PJ, "cost optimal allocation of passenger lines," European Journal of Operational Research, vol. 110, pp. 474-489, 1998.

[8] J. Goossens and C. K. L. Van Hoesel, "a branch and cut approach for solving railway line planning problems," Transportation Science, vol. 38, pp. 379-393, 2004.

[9] T. Lindner, "train schedule optimization in public rail transport. PhD thesis," TU Braunschweig, 2000.

[10] S. Scoll, "customer-oriented line planning, PhD thesis," 2005.

[11] M. Shafia, S. SJ and J. A, "robust train formation planning; proceedings of the IMechE Part F," Journal of Rail and Rapid Transit, vol. 224, pp. 75-90, 2010.

[12] M. Yaghini, A. Alimohammadian and S. S, "a goal programming technique for railroad passenger scheduling," Management Scienece Letters, vol. 2, pp. 535-542, 2012.

[13] M. Yaghini and A. S. S. MAlimohammadian, "a hybrid method to solve railroad passenger scheduling problem," Management Science Letters, vol. 2, pp. 543-548, 2012.

[14] M. Yaghini, A. Alimohammadian and M. Karimi, "optimization of passenger line planning in railway," International Journal of Industrial Engineering and Production Management, vol. 24, pp. 491-501, 2014.

Table 1: Solved problems and proposed solution

No of problem	number of stations	status of offered lines	capacity of each arc	number of decision variables	number of limitation s	branch and bound algorithm		proposed solution	
						target's optimized value	analysis time (sec)	target's optimized value	analysis time (sec)
1	10	F	T	1070	359	1540	420/2	1540	64/7
2	10	F	L	1070	359	2250	378/8	2250	58/5
3	10	H	T	252	138	1540	45/8	1540	10/3
4	10	H	L	252	138	2250	42/1	2250	9/6
5	20	F	T	33190	1819	-	-	5990	2348/9
6	20	F	L	33190	2819	-	-	9500	2153/6
7	20	H	T	7310	1119	5990	6834/8	5990	574/6
8	20	H	L	7310	1119	9500	6151/4	9500	4/502
9	30	F	T	156550	12069	-	-	11050	5049/2
10	30	F	L	156550	12069	-	-	21750	5075/9
11	30	H	T	75120	5733	-	-	11070	4043/1
12	30	H	L	75120	5733	-	-	21750	3762/9